studied in general form in [5], we do not consider these questions here. Figure 2 illustrates the dependence of the Nusselt number Nu (dimensionless convective heat flux through the cavity) on the Rayleigh number Ra for a perfect sphere $s \equiv 0$ (theory:solid line; the region containing the experimental points from [4] is shaded). Near Ra_* theory gives a linear dependence of $Nu - 1$ on $Ra - Ra_*$, which differs from the experimental results in [4].

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EXCITATION OF UNSTABLE WAVES IN BOUNDARY LAYER ON A VIBRATING SURFACE

A. M. Tumin and A. V. Fedorov

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Modern concepts [1] on boundary-layer transition make it possible to develop a computational method for transition Reynolds numbers which includes the analysis of the growth of unstable disturbances in the boundary layer and the determination of the section at which their amplitude initially attains the critical value. Here, in order to develop a closed computational scheme it is necessary to solve the problem of excitation of the so-called Tollmien–Schlichting waves in the boundary layer. In experimental and theoretical studies [2-6], it has been shown that the Tollmien–Schlichting wave can arise due to flow nonuniformities of various types (sharp leading edge of the model, individual roughness element on the surface, localized effect on boundary layer). These results are discussed in sufficient detail in [7]. The adiabatic type excitation mechanism caused by natural, weak flow nonuniformity in the boundary layer on a smooth surface was suggested in [8]. A comprehensive qualitative and quantitative analysis of different types of excitation of Tollmien–Schlichting waves is necessary to solve applied problems. The present paper considers the excitation of unstable waves in boundary layer on a vibrating surface. The formulation of such a problem is discussed in [1].

Problem Formulation. Consider a two-dimensional incompressible boundary layer. Small differences arising from compressibility will be shown later. The coordinate system chosen is: x, distance from the leading edge of the model, downstream along the surface; y, distance normal to the surface; the reference scales are: certain length x_0 for the coordinate x, $\sqrt{\nu x_0/U_0}$ for y, where ν is the coefficient of kinematic viscosity, U_0 is the characteristic free stream velocity. Time is defined in units of $\sqrt{\nu x_0}/U_0^{3/2}$, pressure in terms of $\rho_0 U_0^2$, where ρ_0 is the density. Assume that the mean flow is weakly nonuniform in the absence of disturbances, i.e., for the streamwise and normal components of velocity U and V*, respectively, there are relations $U = U(x, y)$, $V^* = \varepsilon V(x, y)$, $\varepsilon = Re^{-1} = \sqrt{\nu/U_0 x_0} \ll 1$. Linearized Navier–Stokes equations after Fourier transformation in time are written in the form [8]

$$\frac{\partial \mathbf{A}}{\partial y} - H_1\mathbf{A} = \varepsilon H_2 \frac{\partial \mathbf{A}}{\partial x} + \varepsilon H_3 \mathbf{A}, \qquad (1)$$

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where A(x, y) is a four component vector function: A_1 is the fluctuation in the x component of velocity, A_2 is the pressure fluctuation, A_3 is the fluctuation in the y component of velocity, and $A_4 = \partial A_1/\partial y - \varepsilon \partial A_3/\partial x$:

$$H_1 = \begin{Vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & i\omega & 0 \\ 0 & 0 & 0 & 0 \\ -i\omega \mathrm{Re} & 0 & \mathrm{Re}\, U' & 0 \end{Vmatrix}; \quad H_2 = \begin{Vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & -U & \mathrm{Re}^{-1} \\ -1 & 0 & 0 & 0 \\ \mathrm{Re}\, U & \mathrm{Re} & 0 & 0 \end{Vmatrix},$$

where ω is the disturbance frequency; $U' = \partial U/\partial y$. The matrix H_3 in Eq. (1) contains only terms proportional to $\partial U/\partial x$, V, $\partial V/\partial y$, and is associated with weak nonparallelness of the boundary layer flow. Assume that at a certain section the initial conditions are specified in the form of a vector function

$$\mathbf{A}(x_1, y) = \mathbf{A}_0(y). \tag{2}$$

Vibration of the surface in the analyzed segment will be simulated in the form of a small amplitude running wave. For the Fourier harmonics under consideration the equation for the surface $y_w(x)$ is represented as $y_w = a\exp[i\alpha_0(x - x_1)/\varepsilon$, where $\alpha_0 > 0$; x_1 is a certain fixed coordinate. Our choice of reference scales corresponds to the wave number α_0 measured in units of $\sqrt{U_0/\nu x_0}$ and the wavelengths are of the order of boundary layer thickness and are small compared to x_0. The no slip condition at the surface must be satisfied for the disturbed flow. Since the amplitude of vibration is small compared to its wavelength, we get

$$\begin{gathered} U'_w y_w(x)\, e^{-i\omega t} + A_1(x, 0)\, e^{-i\omega t} = O(a^2), \\ A_3(x, 0)\, e^{-i\omega t} = \frac{\partial}{\partial t}[y_w(x)\, e^{-i\omega t}] + O(a)^2, \quad U'_w = \frac{\partial U}{\partial y}(x, 0). \end{gathered} \tag{3}$$

Thus, boundary conditions for the system (1) take the form

$$\begin{gathered} A_1(x, 0) = -aU'_w \exp[i\alpha_0(x - x_1)/\varepsilon], \\ A_3(x, 0) = -i\omega a \exp[i\alpha_0(x - x_1)/\varepsilon]. \end{gathered} \tag{3a}$$

As $y \to \infty$ we assume boundness of the solution:

$$|A_j| < \infty, \; j = 1, \ldots, 4. \tag{4}$$

The problem formulated in (1)-(4), generally speaking, is incorrect. Hence, the initial conditions A_0 are supplemented by conditions that they permit solution with finite growth rate. Actually this requirement for regularization lends to the assumption that the initial conditions are orthogonal to the eigenfunctions of the linearized Navier–Stokes equations, related to the disturbances propagating upstream, and such solutions are not considered in further analysis [9, 10].

Biorthogonal Vector System. The solution to the problem (1)-(4) for the case when the mean flow is weakly nonuniform along x will be expressed in the form of a series of biorthogonal vector systems for the locally homogeneous problem $\{A_\alpha(x, y), B_\alpha(x, y)\}$, formulated in [8, 9].

$$\begin{gathered} \partial \mathbf{A}_\alpha/\partial y - H_1\mathbf{A}_\alpha = i\alpha H_2 \mathbf{A}_\alpha, \\ y = 0, \; A_{\alpha 1} = A_{\alpha 3} = 0, \; y \to \infty, \; |A_{\alpha j}| < \infty, \; j = 1, \ldots, 4; \end{gathered} \tag{5}$$

$$\begin{gathered} \frac{\partial \mathbf{B}_\alpha}{\partial y} + H_1^* \mathbf{B}_\alpha = i\tilde{\alpha} H_2^* \mathbf{B}_\alpha, \\ y = 0, \; B_{\alpha 2} = B_{\alpha 4} = 0, \; y \to \infty, |B_{\alpha j}| < \infty, \; j = 1, \ldots, 4, \end{gathered} \tag{6}$$

where * refers to transposed and complex-conjugate matrix; ∿ refers to complex conjugate; the index α indicates that the eigenvector function corresponds to the eigenvalue α. The systems of Eqs. (5) and (6) depend on x as a parameter. They have four linearly independent solutions whose asymptotic dependence on y outside the boundary layer is expressed in the form

$$\mathbf{z}_1 \sim e^{-\alpha y},\ \mathbf{z}_2 \sim e^{\alpha y},\ \mathbf{z}_3 \sim e^{\lambda y},\ \mathbf{z}_4 \sim e^{-\lambda y},\ \lambda = \sqrt{\alpha^2 + i\mathrm{Re}(\alpha - \omega)}.$$

For definiteness, we choose the branch Re $(\lambda) < 0$. The system of Eqs. (5) and (6) allows four types of solutions from the continuous spectra and one from the discrete spectrum [8, 9]. This discrete spectrum corresponds to Tollmien–Schlichting waves for which the solution can be written in the form of a linear combination $A_{TS} = C_1 z_1 + C_3 z_3$. Here $\alpha_{TS}(x)$ is obtained from the dispersion relation

$$E_{13}(\alpha_{TS}) = (z_{11}z_{33} - z_{13}z_{31})_{y=0} = 0,$$

where z_{ij} denotes the i^{-th} component of the j^{-th} vector. In the continuous spectrum there are waves with $\alpha = \pm ik,\ \alpha = -\frac{i\,\mathrm{Re}}{2}\left[1 \pm \sqrt{1 + \frac{4k^2}{\mathrm{Re}^2} - \frac{4i\omega}{\mathrm{Re}}}\right]$, where k is an arbitrary positive number [8, 9]. Two of these correspond to waves propagating upstream and having exponential growth as $x \to \infty$. The following orthogonal relation exists:

$$\langle H_2\mathbf{A}_\alpha, \mathbf{B}_\beta\rangle = \Delta_{\alpha\beta},\quad \langle \mathbf{A}, \mathbf{B}\rangle = \int_0^\infty (\mathbf{A}, \mathbf{B})\,dy,\quad (\mathbf{A}, \mathbf{B}) = \sum_{j=1}^4 A_j\tilde{B}_j,$$

where $\Delta_{\alpha\beta}$ is the Kronecker delta, where one of the numbers (α, β) belongs to the discrete spectrum; $\Delta_{\alpha\beta} = \delta(\alpha-\beta)$ is the delta function when both the numbers (α, β) belong to the continuous spectrum [8, 9].

In order to develop the solution to the system (1) in the form of a series in terms of the vector A_α, it is necessary to supplement (5) and (6) by the nonhomogeneous solution A_v at y = 0. Outside resonance ($\alpha_0 \neq \alpha_{TS}$) we obtain the vector $A_v(x, y)$ using the equation

$$\partial\mathbf{A}_v/\partial y - H_1\mathbf{A}_v = i\alpha_0 H_2\mathbf{A}_v, \tag{7}$$
$$y = 0,\quad A_{v1} = -aU'_w,\quad A_{v3} = -ia\omega,\quad y\to\infty,\quad |A_{vj}|\to 0,\quad j = 1, \ldots, 4.$$

The solution to Eq. (7) can be written in the form

$$\mathbf{A}_v = \frac{a}{E_{13}}\left[\left(z_{13}i\omega - U'_w z_{33}\right)\mathbf{z}_1 + \left(z_{31}U'_w - i\omega z_{11}\right)\mathbf{z}_3\right], \tag{8}$$

where z_{ij} are computed at y = 0. It is easy to show the existence of the relation

$$\langle H_2\mathbf{A}_v, \mathbf{B}_\alpha\rangle i(\alpha_0 - \alpha) + (\mathbf{A}_v, \mathbf{B}_\alpha)_{y=0} = 0, \tag{9}$$

where α belongs to one of the spectra of the solutions (5), (6).

Procedure for Solving the System (1). The solution to the problem (1)-(4) is sought in the form

$$\mathbf{A}(x, y) = \sum_\alpha{}' c_\alpha(x)\,\mathbf{A}_\alpha(x, y) + \mathbf{A}_v(x, y)\exp\left(i\int_{x_1}^\infty \frac{\alpha_0}{\varepsilon}\,dx\right), \tag{10}$$

where Σ' denotes summation along the discrete spectrum and integration along the continuous spectrum. Substituting (10) in (1) it is possible to obtain a system of equations for the determination of $c_\alpha(x)$, for which the initial value of $c_\alpha(x_1)$ is determined from A_0:

$$c_\alpha(x_1) = \langle H_2(\mathbf{A}_0 - \mathbf{A}_v), \mathbf{B}_\alpha\rangle_{x=x_1}. \tag{11}$$

The excitation of Tollmien–Schlichting waves as a result of the vibrating surface is of interest to us. As we shall see below, it will be extremely intensive because of the presence of resonance point x_*, where $\alpha_0 = \alpha_{TS}(x_*)$. Of all the formulations of the problem on the generation of Tollmien–Schlichting waves in the boundary layer on a vibrating surface we shall consider the resonance case when x_* is the point at which the flow becomes unstable. This is so because, all other factors being the same, the greatest amplification rates are obtained for waves excited in the neighborhood of $x = x_*$. As indicated by computations in the case of flat plates in incompressible flow [11] (see also the numerical example in compressible flow) there is a strong damping of Tollmien–Schlichting waves in the interval from the leading edge to the point of instability. Hence, if the initial section is chosen quite close to the leading edge and the initial amplitude $c_{TS}(x_1)$ determined from (11) is of the order of unity, then, without a great loss of accuracy, the final result does not change if we put $c_{TS}(x_1) = 0$. The amplitude $c_{TS}(x)$ is expressed in the form

$$c_{\mathrm{TS}}(x) = q(x) K(x) \exp\left[i \int_{x_1}^{x} W(x)\, dx\right], \quad K(x) = \exp\left[i \int_{x_1}^{x} \frac{\alpha_{\mathrm{TS}}}{\varepsilon}\, dx\right], \tag{12}$$

$$iW(x) = -\langle H_3 \mathbf{A}_{\mathrm{TS}}, \mathbf{B}_{\mathrm{TS}} \rangle - \left\langle H_2 \frac{\partial \mathbf{A}_{\mathrm{TS}}}{\partial x}, \mathbf{B}_{\mathrm{TS}} \right\rangle.$$

Then for q(x) we have

$$\frac{dq}{dx} = -\left[\left\langle H_2 \frac{\partial \mathbf{A}_v}{\partial x}, \mathbf{B}_{\mathrm{TS}} \right\rangle + \langle H_3 \mathbf{A}_v, \mathbf{B}_{\mathrm{TS}} \rangle\right] \exp[i\theta(x)],$$

$$\theta(x) = \int_{x_1}^{x} \left[\frac{(\alpha_0 - \alpha_{\mathrm{TS}})}{\varepsilon} - W(x)\right] dx. \tag{13}$$

Solving Eq. (13) with zero initial conditions, we write

$$\mathbf{A} = \mathbf{A}_v \exp\left[i \int_{x_1}^{x} \frac{\alpha_0}{\varepsilon}\, dx\right] - \int_{x_1}^{x} \left[\left\langle H_2 \frac{\partial \mathbf{A}_v}{\partial x}, \mathbf{B}_{\mathrm{TS}} \right\rangle + \langle H_3 \mathbf{A}_v, \mathbf{B}_{\mathrm{TS}} \rangle\right] e^{i\theta} dx\, e^{i\theta_{\mathrm{TS}}} \mathbf{A}_{\mathrm{TS}}, \tag{14}$$

$$\theta_{\mathrm{TS}}(x) = \int_{x_1}^{x} \left(\frac{\alpha_{\mathrm{TS}}}{\varepsilon} + W\right) dx.$$

Solution of (8) in the neighborhood of the resonance point is not suitable because at $x = x_*$ the function $E_{13}(\alpha_0)$ in (8) becomes zero. When there is resonance ($\alpha_0 = \alpha_{TS}$) the solution of the problem with nonhomogeneous boundary conditions at y = 0 should necessarily contain simultaneously A_{TS} and a certain vector function which ensures the fulfillment of the boundary conditions at y = 0. It was because of this situation that in the analysis we wrote the solution A (14) as a summation since it is then uniformly valid for all x.

For the sake of convenience, we introduce the vector Q(x, y), which is regular at $x = x_*$ and determined from the equation

$$\mathbf{A}_v = \mathbf{Q}(x, y)/(x - x_*). \tag{15}$$

Direct computations show that at $x = x_*$ the conditions $Q_1(x_*, 0) = Q_3(x_*, 0) = 0$ are satisfied, i.e., at $x = x_* Q \sim A_{TS}$ without a loss of generality we choose normalization of A_{TS} such that $Q(x_*, y) = A_{TS}(x_*, y)$. The choice for the normalization of B_{TS} can always be ensured by the equation

$$\langle H_2 \mathbf{A}_{\mathrm{TS}}, \mathbf{B}_{\mathrm{TS}} \rangle = 1. \tag{16}$$

The singular point that depends on "slowly" varying x is separated from the integral expression in (14). For this purpose consider the expansion as $x \to x_*$:

$$\left[\left\langle H_2\frac{\partial \mathbf{A}_v}{\partial x}, \mathbf{B}_{TS}\right\rangle + \langle H_3\mathbf{A}_v, \mathbf{B}_{TS}\rangle\right]\exp\left[-i\int\limits_{x_1}^{x} W dx\right] =$$

$$= \left[\frac{\left\langle H_2\frac{\partial \mathbf{Q}}{\partial x}, \mathbf{B}_{TS}\right\rangle}{x - x_*} - \frac{\langle H_2\mathbf{Q}, \mathbf{B}_{TS}\rangle}{(x-x_*)^2} + \frac{\langle H_3\mathbf{Q}, \mathbf{B}_{TS}\rangle}{x - x_*}\right]\exp\left[-i\int\limits_{x_1}^{x} W dx\right] \simeq \tag{17}$$

$$\simeq \left[-\frac{\langle H_2\mathbf{Q}, \mathbf{B}_{TS}\rangle_*}{(x-x_*)^2} - \frac{\left\langle \frac{\partial H_2}{\partial x}\mathbf{Q}, \mathbf{B}_{TS}\right\rangle_*}{x-x_*} - \frac{\left\langle H_2\mathbf{Q}, \frac{\partial \mathbf{B}_{TS}}{\partial x}\right\rangle_*}{x-x_*} + \frac{\langle H_3\mathbf{Q}, \mathbf{B}_{TS}\rangle_*}{x-x_*} + \right.$$

$$\left. + \frac{\langle H_2\mathbf{Q}, \mathbf{B}_{TS}\rangle_*}{x-x_*} iW(x_*) + O(1)\right]\exp\left[-i\int\limits_{x_1}^{x_*} W(x)\, dx\right],$$

where the index * denotes that the scalar product is computed at $x = x_*$. Using (16) and W from (12), we find that the expression (17) gives the asymptote

$$\sim -\exp\left[-i\int\limits_{x_1}^{x_*} W dx\right]\Big/(x-x_*)^2 + O(1):$$

$$\mathbf{A}(x, y) = \left[F(x, \varepsilon) + \frac{\exp\left(-i\int\limits_{x_1}^{x_*} W dx\right)}{x_1 - x_*}\right]e^{i\theta_{TS}}\mathbf{A}_{TS}$$

$$+ \frac{\left[\mathbf{Q}(x, y) - \mathbf{A}_{TS}\exp\left(+i\int\limits_{x_*}^{x} W dx\right)\right]}{x - x_*}\exp\left(i\int\limits_{x_1}^{x}\frac{\alpha_0}{\varepsilon}dx\right) + D(x, \varepsilon)\,\mathbf{A}_{TS}e^{i\theta_{TS}}, \tag{18}$$

$$F(x, \varepsilon) = -\int\limits_{x_1}^{x}\left[\left(\left\langle H_2\frac{\partial \mathbf{A}_v}{\partial x}, \mathbf{B}_{TS}\right\rangle + \langle H_3\mathbf{A}_v, \mathbf{B}_{TS}\rangle\right)\exp\left(-i\int\limits_{x_1}^{x} W dx\right) + \right.$$

$$\left. + \frac{\exp\left(-i\int\limits_{x_1}^{x_*} W dx\right)}{(x-x_*)^2}\right]\exp\left[i\int\limits_{x_1}^{x}\frac{(\alpha_0 - \alpha_{TS})}{\varepsilon}dx\right]dx,$$

$$D(x, \varepsilon) = i\int\limits_{x_1}^{x}\frac{(\alpha_0 - \alpha_{TS})}{\varepsilon(x-x_*)}\exp\left[i\int\limits_{x_1}^{x}\frac{(\alpha_0 - \alpha_{TS})}{\varepsilon}dx\right]dx\exp\left(-i\int\limits_{x_1}^{x_*} W dx\right).$$

It is possible to ascertain that the solution (18) is regular at $x = x_*$ and satisfies the nonhomogeneous boundary conditions at y = 0 for all $x \neq x_*$. Since A_{TS} is determined with homogeneous boundary conditions at y = 0 for all x, we get $\partial A_{TS1}/\partial x = \partial A_{TS3}/\partial x = 0$ at y = 0. Consequently, in order to show that the solution (18) satisfies boundary conditions at y = 0 and at the point $x = x_*$, it is necessary to consider the expansion of (18) in the neighborhood of x_* and ascertain that the term with $\partial Q(x_*, 0)/\partial x$ ensures the satisfaction of boundary conditions even at $x = x_*$. This verification is carried out by direct computations with the determination of Q in (15), expressions (8) for A_v, taking into account that $E_{13}(\alpha_0) = 0$ at $x = x_*$. The solution to the problem (1)-(4) is thus obtained.

Asymptotic Estimate of the Amplitude of the Tollmien—Schlichting Wave. The integrands in F and D from (18) contain the factor $\exp\left(i\int\limits_{x_1}^{x}\frac{\alpha_0 - \alpha_{TS}}{\varepsilon}dx\right)$, which makes it possible to obtain an asymptotic estimate as $\varepsilon \to 0$. In the neighborhood of $x = x_*$ we have

$$i(\alpha_0 - \alpha_{TS}) = -i\frac{d\alpha_{TS}}{dx}(x_*)(x - x_*) + O[(x-x_*)^2],$$

where $\text{Real}\left(i\frac{d\alpha_{TS}}{dx}\right) > 0$. It is easy to obtain at $x - x_* >> \sqrt{\varepsilon}$ [12] that $F \sim \sqrt{\varepsilon}$, $D \sim 1/\sqrt{\varepsilon}$, i.e., the basic contribution to the amplitude of the Tollmien—Schlichting wave in solution (18) is made by the term with the factor $D(x, \varepsilon)$ which attains its value $\sim 1/\sqrt{\varepsilon}$ in the neighborhood of x_*. Using (9) as $x \to x_*$, we get

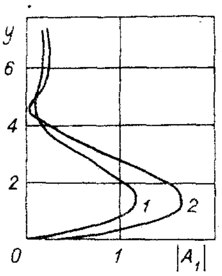

Fig. 1

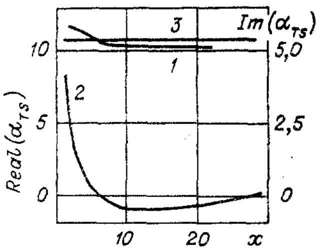

Fig. 2

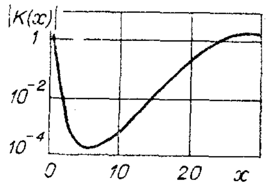

Fig. 3

TABLE 1

$f\cdot 10^6$	Re	$\alpha_0\cdot 10^2$	$\lvert q_m\rvert/a$
20	962	6,45	2,14
40	645	7,85	1,77
60	515	8,89	1,57

$$i\frac{d\alpha_{\mathrm{TS}}}{dx}(x_*) = -a\left(U'_w B_{\mathrm{TS}1} + i\omega B_{\mathrm{TS}3}\right)_{y=0,x=x_*}.$$

Then the basic contribution to (18) as $x - x_* \gg \sqrt{\varepsilon}$, describing the excited Tollmien–Schlichting wave can be expressed in the form

$$g\exp\left[i\int_{x_*}^{x}\left(\frac{\alpha_{\mathrm{TS}}}{\varepsilon}+W\right)dx\right]\mathbf{A}_{\mathrm{TS}},$$

where the amplitude g is determined using the transfer method [12]. For $|g|/a$, we get the expression

$$\frac{|g|}{a} = \frac{1}{\sqrt{\varepsilon}}\sqrt{\frac{2\pi}{\left|\frac{d\alpha_{\mathrm{TS}}}{dx}(x_*)\right|}}\left(U'_w B_{\mathrm{TS}1} + i\omega B_{\mathrm{TS}3}\right)_{y=0,x=x_*}. \qquad (19)$$

Compressibility Considerations. Numerical Example. Consideration of compressible flows leads only to a change in the concrete form of the matrices H_1, H_2, and H_3 in (1) and the determination of the vector A. In the formulation of the boundary conditions at y = 0 in (5) and (7) for the flow past a surface made of highly heat-conducting material, it is necessary to ensure that the temperature fluctuation should be zero. In this case the fundamental system of solution for the (5) and (6) will consist of six linearly independent vectors [13], that leads to the change in the concrete form of A_V in (8). Types of eigensolutions to (5) for compressible flows are considered in [10]. However, the complete asymptotic analysis remains as before and the result (19) remains true if we keep in view that the first and the third components of the vector A correspond to disturbances of the longitudinal and normal velocity components. In the present work a numerical computation has been carried out for the case of a vibrating surface of an insulated flat plate at Prandtl number 0.72, stagnation temperature 310°K, and free stream Mach number 0.6, assuming that the coefficient of viscosity depends on temperature according to Sutherland's formula. Figure 1 shows the dependence of $|A_{TS1}|$ and $|A_{V_1}|$ on y at Re = 800 and nondimensional frequency parameter $f = \omega\mu_0/\rho_0 U_0^2 = 20\cdot 10^{-6}$ (μ_0 is the coefficient of viscosity in the free stream; curves 1, 2 respectively). The computation corresponds to x = 1, a = 1.2, and $\partial A_{TS_1}/\partial y = 2$ at y = 0. Figure 2 shows Real (α_{TS}) and Im (α_{TS}) (curves 1, 2, respectively) for the frequency parameter $f = 20\times 10^{-6}$ and Re = 400, and also the dependence of α_0 on x (curve 3). Figure 3 shows the dependence of $|K|$ from (12) for the same values of parameters at $x_1 = 0.42$ which confirms the above mentioned considerations on the possibility of neglecting $c_{TS}(x_1)$ (Tollmien–Schlichting wave excited in the neighborhood of the leading edge).

Expression (19) for the amplitude of excited Tollmien–Schlichting wave is not invariant with respect to the choice of normalization of eigenfunctions A_{TS} and B_{TS}. In order to obtain the invariant value, the expression (19) has to be multiplied by $|F_{TS}/\langle H_2 A_{TS}, B_{TS}\rangle|$, where F_{TS} is the amplitude of disturbance of the physical quantity of interest to us and computed from the components of the eigenvector A_{TS}. In the present paper the x component of the fluctuation in mass flux near its maximum was chosen as F_{TS}. Computed results for various frequency parameter f are given in Table 1. Also shown are the values of the parameter Re corresponding to the choice of $x_0 = x_*$ and the resonant values of α_0. The quantity q_m is equal to the amplitude of disturbance of the longitudinal component of mass flow in the neighborhood of its maximum for the excited Tollmien–Schlichting wave.

In the present paper the terms $\sim O(\alpha^2)$ were neglected in formulating the boundary conditions and it was assumed that the velocity and temperature profiles of the mean flow coincided with the corresponding profiles in the absence of vibration. This question was treated in detail using asymptotic techniques in [14], where it was shown that such an approximation is true for $\alpha \ll \delta_n$, where $\delta_n = (\omega Re)^{-1/2}$ is the thickness of the viscous wall layer. For such a limitation the inequality $c_{TS} \ll 1$ will be satisfied and it agrees with the linear formulation of the given problem.

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